

AP Calculus Summer Packet

ANSWERS

Simplify using only positive exponents.

1. -5^{-x}

$-\frac{1}{5^x}$

2. $\frac{\frac{1}{2}(2x+5)^{-3/2}}{\frac{3}{2}}$

$\frac{1}{3(2x+5)^{3/2}}$

3. $2\left(\frac{2}{2-x}\right)\left(\frac{-2}{(2-x)^2}\right)$

$\frac{-8}{(2-x)^3}$

4. $(16x^2y)^{3/4}$

$8x^{3/2}y^{3/4}$

5. $-\left(\frac{x^{1/2}}{2}\right)\sin\sqrt{x}$

$-\frac{x^{1/2}\sin x^{1/2}}{2}$

6. $\frac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$

$\frac{(4(x-4))^{1/2}}{(x-4)^{3/4}} = \frac{2(x-4)^{1/2}}{(x-4)^{3/4}} = \frac{2}{(x-4)^{1/4}}$

Find the domain of the following functions. Make sure to use interval notation.

7. $y = \frac{x^2-4}{2x+4} = 0$

$2x = -4$
 $x = -2$ ← "hole"

Domain: \mathbb{R} except $x = -2$

ie. $(-\infty, -2) \cup (-2, \infty)$

8. $y = \frac{x^2+8x+12}{\sqrt[4]{x+5}} = 0$ ← note denominator must be positive

$x+5 = 0$
 $x = -5$

Domain: $(-5, \infty)$

Factor completely.

9. $x^5 + 11x^3 - 80x$

$x(x^4 + 11x^2 - 80)$
 $x(x^2 + 16)(x^2 - 5)$

10. $2x^2 + 50y^2 - 20xy$

$2(x^2 + 25y^2 - 10xy)$
 $2(x^2 - 10xy + 25y^2)$
 $2(x-5y)(x-5y)$
 $2(x-5y)^2$

11. $(x-3)^2(2x+1)^3 + (x-3)^3(2x+1)^2$

$(x-3)^2(2x+1)^2 \overset{GCF}{\left[(2x+1) + (x-3) \right]}$
 $(x-3)^2(2x+1)^2(3x-2)$

Describe, in words, the transformations that would take place to $f(x)$ in each of the follow.

12. $f(x) - 3$

- down 3

13. $f(x-3)$

- right 3

14. $-f(x+5)$

- reflection over x-axis
- left 5

15. $6f(x) + 3$

- vertical stretch
- up 3

16. $f(2x)$

- horizontal shrink

Solve each equation by factoring, graphing, or using the quadratic formula.

17. $7x^2 - 3x = 0$

$$x(7x - 3) = 0$$

$$x = 0, \frac{3}{7}$$

18. $4x(x-2) - 5x(x-1) = 2$

$$4x^2 - 8x - 5x^2 + 5x - 2 = 0$$

$$-x^2 - 3x - 2 = 0$$

$$x^2 + 3x + 2 = 0$$

$$(x+2)(x+1) = 0$$

$$x = -2, -1$$

19. $x^2 + 6x + 4 = 0$

$$-b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-6 \pm 2\sqrt{5}}{2}$$

$$x = -3 \pm \sqrt{5}$$

20. $2x^2 - (x+2)(x-3) = 12$

$$2x^2 - (x^2 - x - 6) - 12 = 0$$

$$2x^2 - x^2 + x + 6 - 12 = 0$$

$$x^2 + x - 6 = 0$$

$$(x+3)(x-2) = 0$$

$$x = -3, 2$$

21. $x + \frac{1}{x} = \frac{13}{6}$

$$\frac{x^2 + 1}{x} = \frac{13}{6}$$

$$6x^2 + 6 = 13x$$

$$6x^2 - 13x + 6 = 0$$

$$(3x-2)(2x-3) = 0$$

$$x = \frac{2}{3}, \frac{3}{2}$$

22. $x - 10\sqrt{x} = -9$

$$x - 10\sqrt{x} + 9 = 0$$

$$(\sqrt{x} - 9)(\sqrt{x} - 1) = 0$$

$$x = 81, 1$$

Find the equations of all vertical ($x=?$) and horizontal ($y=?$) asymptotes (if they exist).

23. $y = \frac{x}{x+4}$

$$HA: y = 1$$

$$VA: x = -4$$

24. $y = \frac{x+5}{x^2-1}$

$$HA: \text{none}$$

$$VA: x = \pm 1$$

25. $y = \frac{x+4}{x^2+9}$

$$HA: y = 0$$

$$VA: \text{none}$$

Simplify the following.

26. $\frac{x}{\frac{2}{x} \cdot x - \frac{1}{2}}$

$$= \frac{x}{\frac{2x-1}{2}}$$

$$= \frac{2x}{2x-1}$$

27. $\frac{\frac{3}{x} \cdot x - \frac{1}{x}}{\frac{2}{x} \cdot x + \frac{1}{x}}$

$$= \frac{x^2 - 1}{\frac{x^2 + 1}{x}}$$

$$= \frac{x(x^2 - 1)}{x(x^2 + 1)}$$

$$= \frac{x^2 - 1}{x^2 + 1}$$

28. $\frac{x^{-3} - x}{x^2 - 1} = \frac{\frac{1}{x^3} - x \cdot \frac{x^3}{x^3}}{\frac{1}{x^2} - 1 \cdot \frac{x^2}{x^2}}$

$$= \frac{1 - x^4}{x^3 \cdot \frac{1 - x^2}{x^2}}$$

$$= \frac{x^2(1 - x^4)}{x^3(1 - x^2)} = \frac{x^2(1 - x^2)(1 + x^2)}{x^3(1 - x^2)}$$

$$= \frac{1 + x^2}{x}$$

If $f(x) = x^2$, $g(x) = 2x - 1$, $h(x) = 2^x$, find the following.

29. $f(g(2))$

$$f(2(2) - 1)$$

$$f(3)$$

$$(3)^2$$

$$9$$

30. $g(f(2))$

$$g((2)^2)$$

$$g(4)$$

$$2(4) - 1$$

$$7$$

31. $g(f(h^{1/2}))$

$$g(f(2^{1/2}))$$

$$g(f(\sqrt{2}))$$

$$g((\sqrt{2})^2)$$

$$g(2)$$

$$2(2) - 1$$

Solve for x.

$$32. \frac{2^2}{2^3} - \frac{5}{6} = \frac{1}{x}$$

$$\frac{4}{6} - \frac{5}{6} = \frac{1}{x}$$

$$\frac{-1}{6} = \frac{1}{x}$$

$$-x = 6$$

$$x = -6$$

Solve for x on the interval $[0, 2\pi)$.

$$35. \sin x = \frac{1}{2}$$

$$x = \pi/6, 5\pi/6$$

$$33. x + \frac{6}{x} = 5$$

$$\frac{x^2 + 6}{x} = 5$$

$$5x = x^2 + 6$$

$$x^2 - 5x + 6 = 0$$

$$(x-3)(x-2) = 0$$

$$x = 3, 2$$

$$34. \frac{x-5}{x+1} = \frac{3}{5}$$

$$5x - 25 = 3x + 3$$

$$2x = 28$$

$$x = 14$$

$$36. 2 \sin x \cos x + \sin x = 0$$

$$\sin x (2 \cos x + 1) = 0$$

$$\downarrow$$

$$x = 0, \pi$$

$$\downarrow$$

$$2 \cos x + 1 = 0$$

$$\cos x = -1/2$$

$$x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$37. \sin^2 x + 4 + \cos^2 x = x$$

$$\sin^2 x + \cos^2 x + 4 = x$$

$$1 + 4 = x$$

$$x = 5$$

Answer the following questions over a variety of topics.

38. Let f be a linear function where $f(2) = -5$ and $f(-3) = 1$. Find $f(x)$ and write in point-slope form.

$$m = \frac{1 - (-5)}{-3 - 2} = -\frac{6}{5}$$

$$y - y_1 = m(x - x_1)$$

$$y + 5 = -\frac{6}{5}(x - 2)$$

OR

$$y - 1 = -\frac{6}{5}(x + 3)$$

39. Find an equation for the line, in point-slope form, that contains $(5, 1)$ and is perpendicular to $6x - 3y = 2$.

$$6x - 3y = 2$$

$$-3y = -6x + 2$$

$$y = 2x - 2/3$$

\uparrow
m

$$\perp m = -1/2$$

$$y - 1 = -1/2(x - 5)$$

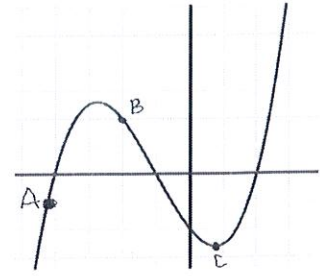
40. Use the table to calculate the average rate of change from $t=1$ to $t=3$.

t	0	1	2	3
x(t)	8	7	5	1

$$m = \frac{1 - 7}{3 - 1} = \frac{-6}{2} = -3$$

41. Order the points A, B, and C, from least to greatest, by their rates of change.

B, C, A



42. Find the points of intersection in the graphs of $(y)^2 = (x-1)^2$ and $y^2 = 2x+6$.

$$\begin{aligned} (x-1)^2 &= 2x+6 \\ x^2 - 4x - 5 &= 0 \\ (x-5)(x+1) &= 0 \\ x &= 5, -1 \end{aligned}$$

43. Rewrite as a single logarithmic expression: $\frac{1}{2} \ln(x-3) + \ln(x+2) - 6 \ln x$.

$$\begin{aligned} &= \ln(x-3)^{1/2} + \ln(x+2) - \ln x^6 \\ &= \ln \left[\frac{\sqrt{x-3} \cdot (x+2)}{x^6} \right] \end{aligned}$$

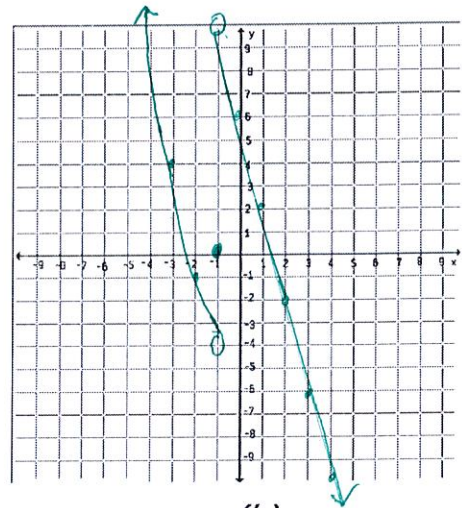
Evaluate the following:

44. $\sin\left(\frac{7\pi}{6}\right) = -\frac{1}{2}$

45. $\sec\left(-\frac{2\pi}{3}\right)$
 $= \frac{1}{\cos(-2\pi/3)}$
 $= \frac{1}{-1/2}$
 $= -2$

46. $\tan\left(\frac{\pi}{2}\right)$
 $= \frac{\sin \pi/2}{\cos \pi/2} = \frac{1}{0} = \text{DNE}$

47. Sketch a graph of the piecewise function $f(x) = \begin{cases} x^2 - 5, & x < -1 \\ 0, & x = -1 \\ 6 - 4x, & x > -1 \end{cases}$

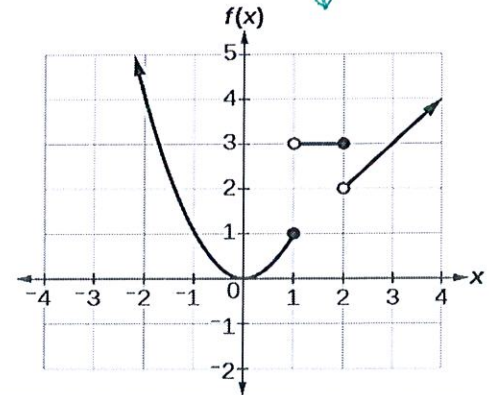


Use the graph of $f(x)$ to the right to find the following.

48. $f(2) = 3$

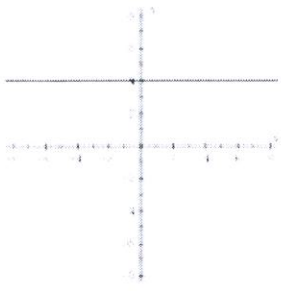
49. $f(x) = 2$
 $x = -1.5$

50. $f(x) = 1$
 $x = \pm 1$

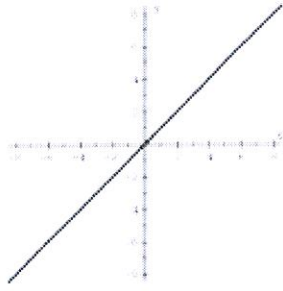


Know all of the following graphs of basic parent functions:

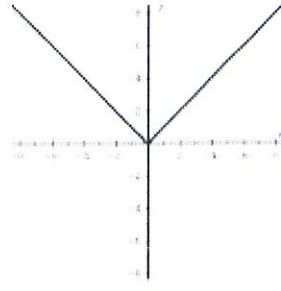
PARENT FUNCTIONS



$f(x) = a$
Constant

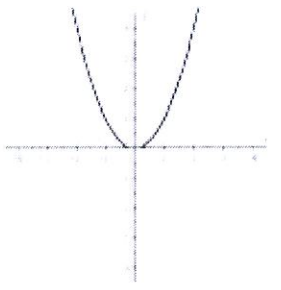


$f(x) = x$
Linear

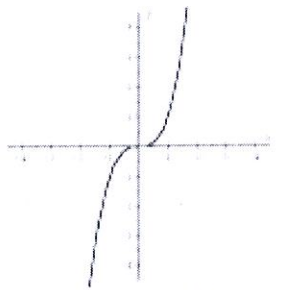


$f(x) = |x|$
Absolute Value

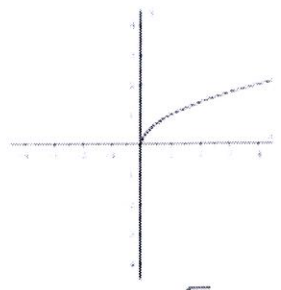
Memorize



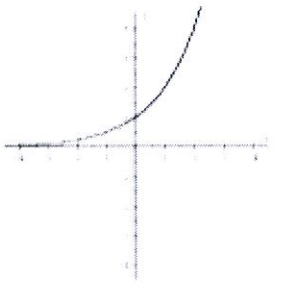
$f(x) = x^2$
Quadratic



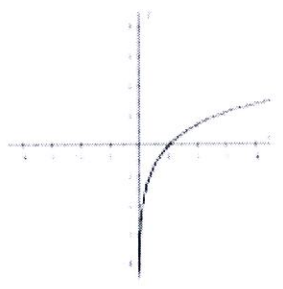
$f(x) = x^3$
Cubic



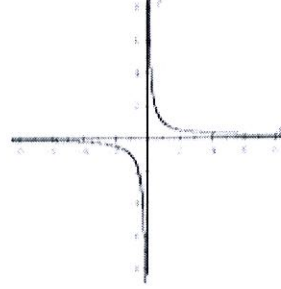
$f(x) = \sqrt{x}$
Square Root



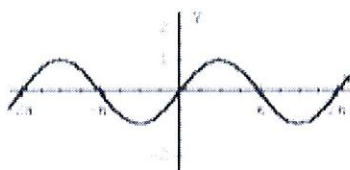
$f(x) = a^x$
Exponential



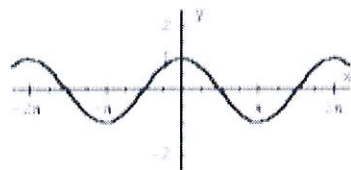
$f(x) = \ln x$
Logarithmic



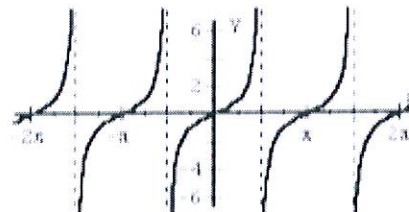
$f(x) = \frac{1}{x}$
Reciprocal



$f(x) = \sin x$



$f(x) = \cos x$
Trigonometric Functions



$f(x) = \tan x$